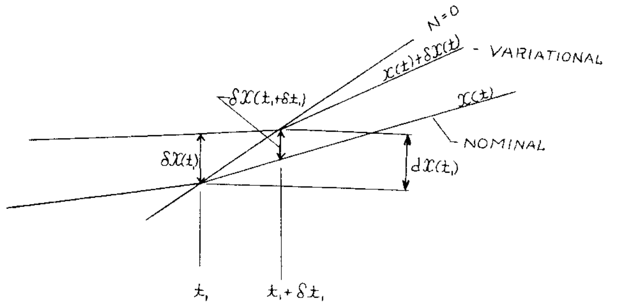

Fig. 1 Definition of variational symbols as used by authors.

Using these expressions, the development of our Eqs. (19–30) in Ref. 1 is relatively straightforward.

Consider next the relabeling of Fig. 1 as indicated by Fig. 2. (This definition of terms corresponds to that used by Wood in private correspondence with the authors.) The use of terms as defined by this figure readily leads to

$$dx(t_1) = \delta x(t_1^-) + \dot{x}(t_1^-)\delta t_1 + \delta\dot{x}(t_1^-)\delta t_1 + \tfrac{1}{2}\ddot{x}(t_1^-)\delta t_1^2 \quad (4)$$

$$dx(t_1) = \delta x(t_1^+) + \dot{x}(t_1^+)\delta t_1 + \delta\dot{x}(t_1^+)\delta t_1 + \tfrac{1}{2}\ddot{x}(t_1^+)\delta t_1^2 \quad (5)$$

Certain other second-order terms that vanish in the ensuing developments have been omitted in Eqs. (2–5) for clarity. Now, if one uses Eqs. (4) and (5), then our Eqs. (22) and (24) are modified as Wood has indicated above.

Note that Eqs. (2) and (3) are identical in form to Eqs. (4) and (5) with the exception that Wood uses $\delta x(t_1^+)$ and $\delta x(t_1^-)$ whereas we only use $\delta x(t_1)$. Wood's results are therefore based on the assumption that $\delta x(t)$ can be discontinuous at t_1. We believe that this is clearly not the case since $\delta x(t)$ is a well defined, continuous, piecewise differentiable function over its interval of definition.

Let us next consider the effect of the "missing terms" in practical problems. The terms at issue are

$$[\dot{x}^T(t_1^-) - \dot{x}^T(t_1^+)]\delta\lambda(t_1^+) \quad (6)$$

and

$$[\dot{x}^T(t_2^-) - \dot{x}^T(t_2^+)]\delta\lambda(t_2^+) \quad (7)$$

Now, in the problem formulation

$$\dot{x} = f[x, u, t]$$

where f is a continuous function of its arguments. Wood has carefully pointed out above that in most practical problems, u is a continuous function of t at the constraint intersection times, t_1 and t_2. Since Eq. (7) clearly implies that x is continuous at t_1 and t_2, it follows that Eqs. (6) and (7) must vanish in the case of a regular Hamiltonian.

Summarizing, we believe that Wood's claim that terms are missing in Eqs. (22) and (24) of Ref. 1 is in error and that, in any case, these terms are usually zero.

There are several typographical errors in Ref. 1. These should be corrected as follows.

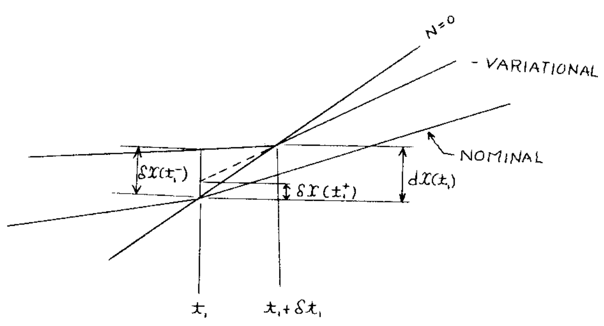

Fig. 2 Definition of variational symbols as used by L. J. Wood.

Equation (9): The $\Omega_x\,\delta x(t_f)$ term inside the first brackets should be $\Omega_x\,\delta x(t_f)\delta t_f$. Also, $\varepsilon_2\delta t_2/2$, should be $\varepsilon_2\delta t_2{}^2/2$.

Equation (14): The x term at the end of the first brackets should be $\dot{x}(t_1^-)$. The $H_u(t_1^+)$ term should be $H_t(t_1^+)$. The S_t^q term should be Z_t.

Equation (15): The $\Omega_x\,\delta x(t_f)\delta t_1$, term should be $\Omega_x\,\delta x(t_f)\delta t_f$.

Equation (41): The Ω term should be $\dot{\Omega}$.

Equation (43): ε_3 should be ε_2.

Equation (51): The equation should read $\dot{R} = -(A^T - SB)R$.

Equation (61): The (1,2) element of the matrix of Eq. (61) should contain the additional term $\varepsilon_1{}^T$.

Sigmas in Eqs. (62, 66, 68, 69, 74, 76, 82, 85, and 86) should be epsilons.

Equation (87): Should read $h(t_1) = \tilde{h}_c(t_1)$.

Equation (125): $x(t)$ should be $\delta x(t)$.

Below Eqs. (92): $\tilde{I} = 1 - I^{-1}bb^T/\beta$; $\tilde{F} = F - db^T/\beta$.

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Comments on "Solution of Nonlinear Problems in Magnetofluiddynamics and Non-Newtonian Fluid Mechanics Through Parametric Differentiation"

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NATH[1] presents an application of the method of parametric differentiation to the solution of boundary layers in magnetofluiddynamics. He gives some results for constant density-viscosity product ($C \equiv \mu\rho/\mu_e\rho_e = 1.0$) flows at stagnation points. The purpose of this Comment is to forestall the possible misunderstanding resulting from Nath's claims by pointing out that the proposed method of solution is ill suited to boundary-layer calculations and that Nath's results bear little resemblance to those obtained when realistic gas properties are used in the calculations.[2,3]

In the method of parametric differentiation, the defining equations are differentiated with respect to some parameter and the resulting equations are integrated numerically. For boundary-layer problems this results in a straightforward successive substitution procedure. By employing degenerate gas properties, Nath avoided the problems of differentiating the nonlinear functions which describe the variation of gas transport properties across the boundary layer, and the strong coupling of the equations which destabilizes the successive substitution technique. It is known[4] that the general variable fluid properties boundary layers can be solved in operator form by a method which employs simple quadratures. Therefore, the use of a cumbersome method whose range of applicability has not been demonstrated is difficult to comprehend.

To illustrate the irrelevance of the degenerate-gas-properties results to the actual problem, Table 1 compares surface shear

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Table 1 Axisymmetric stagnation point shear stress, τ_w, for $Pr = 0.27$

g_o	1.0	0.5	0.1	0.05	0.01	0.001
(a) $C = 1$	0.928	0.835	0.754	0.740	0.740	0.740
(b) $C = g_o^{-1/2}$	0.928	0.749	0.596	0.576	0.559	0.556
(a)/(b)	1.0	0.898	0.790	0.772	0.756	0.751

stress, τ_w, for $C = 1.0$ with that for C varying inversely as the square root of the local temperature. Calculations were performed using the method of Ref. 2.

The results of Nath ($C = 1$) are related to more realistic variable C data through some function of g_o which is not known until the full problem is solved. It is thus seen that Ref. 1 is an exercise in applying a poorly chosen method to unrealistic problems. The present Comment attempts to obviate possible misinterpretations which could tempt fluid dynamicists to follow the course chosen by Nath.[1]

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Reply by Author to W. J. Franks

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In Ref. 1, an application of the method of parametric differentiation to the solutions of boundary-layer equations in magnetofluiddynamics and in non-Newtonian fluid mechanics was presented. The aim of the analysis was to show that the method can be applied successfully to complex flowfields containing a number of parameters. For the first problem, it has been pointed out by Bush[2] and also verified by the present author that the usual method of solving two-point boundary-value problems, i.e., the method of shoot and hunt fails for large value of the magnetic parameter $M(M > 10)$ even under the simplifying assumptions of $Pr = 1$ and constant density–viscosity product, i.e., $c = \rho\mu/\rho_e\mu_e = 1$. In our analysis, we assumed $c = 1$ because we wanted to compare our results with the corresponding results obtained by a shoot and hunt scheme which were available to the author for the case $c = 1$. It may be remarked that the present method of solution is valid even for $M = 100$, but the results for $M > 10$ were not tabulated in Ref. 1 because no comparison could be made.

In fact, for $c = g^{-1/2}$, Nath[3,4] has obtained the solution of the above problem using a shoot and hunt scheme and the method of parametric differentiation. Further, the solution of general three-dimensional stagnation point has been obtained by Vimala[5] using the method of parametric differentiation for $c = g^{-1/2}$ in contrast to $c = 1$ as considered by Libby.[6] The inadequacy of solutions obtained under simplifying assumption of $c = 1$ is well known. However, it is a common practice to use such an assumption.[2,6]

In view of the abovementioned results, the comment of Franks that the method is ill suited for the solutions of the boundary-layer equations is not justified. On the other hand, it can be concluded that the method of parametric differentiation is another powerful technique for solving boundary-layer equations with realistic property variations.

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Errata

A Study of Compressible Potential and Asymptotic Viscous Flows for Corner Region

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Equation (13) should read:

$$U_{1,c} = -\frac{\beta}{2m}\left[\frac{[(x^2+m^2y^2)^{1/2}-x]^{1/2}}{(x^2+m^2y^2)^{1/2}} + \frac{[(x^2+m^2z^2)^{1/2}-x]^{1/2}}{(x^2+m^2z^2)^{1/2}}\right] \quad (13)$$

The last paragraph in the subsection "Subsonic Flow" should read:

For the compressible flow velocities given by Eqs. (13–15), the quantity β has the same significance as expressed in Eq. (5), although its numerical value will be, in general, different from that for incompressible flow.

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